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WAVE DISTURBANCE OF THE TEMPERATURE FIELD IN LOW-THERMAL-CONDUCTIVITY MATERIALS

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It has been established that under conditions where the heat flux supplied exceeds the level permissible for heating a material without the destruction of its surface, the velocity of propagation of heat is independent of the prehistory of heating and is determined by the steady-state value of the temperature on the surface being destroyed. The possibility of disturbing the temperature field in a wave manner has been shown; the wave disturbance is attenuated with increase in the heating time, if there is no removal of mass, and reduces the time in which the stationary regime of destruction of the surface in mass removal is reached. An analytical expression for calculation of the wave disturbance of the temperature field has been proposed.

Fluctuations, dissipative structures, and self-organization are of considerable importance in high-temperature technologies of production of new materials [1]. In any dissipative system, which is initially in a homogeneous stationary state, ordered inhomogeneous states, i.e., dissipative structures, must appear under the action of constant fluxes, including heat ones, whose intensity exceeds a certain critical value. These structures differ from those in equilibrium in that their existence requires the inflow of energy from the outside, since their self-organization is associated with the exchange of energy and a substance with the environment [2]. Reaching the instability threshold, the initially stable dissipative structure begins to oscillate, and fluctuations appearing in it give rise to the self-organization of a new dissipative structure more stable at a given energy level. This structure may be maintained as long as the energy fluxes are preserved. Vibrational processes appearing in transition of the structure to a new state have been found in chemistry (Belousov–Zhabotinskii reaction), in hydrodynamics (formation of vortices in laminar-to-turbulent transition), in optics, etc.

High-temperature technologies make it possible to create materials which are in a strongly nonequilibrium state. Instabilities and structurization of different kinds have been noted to be processes common with materials science. In high-temperature heating (lasers, solar-energy concentrators, plasmatrons, and others), the surface is often heated not only to the melting point but to the evaporation temperature as well. Under such conditions, the character of propagation of heat may substantially influence the properties of the materials produced, and study of the regularities of its propagation is a topical problem.

By a dynamically stable dissipative structure in the process of stationary thermal destruction of a material we mean a heated layer moving with a constant velocity under the action of the energy supplied to the surface under heating [3]. Such a structure satisfies all the conditions enumerated above; a continuously supplied heat flux exceeding the value for which the destruction (melting) of the material surface begins is mainly responsible for the formation of this structure.

However, it becomes necessary to explain the manner in which the fundamental property of nonequilibrium systems, which manifests itself as the capacity for transforming into an ordered state as a result of fluctuation, is realized for this structure (dynamically heated layer) [1]. It has been assumed until the present time that no disturbances of a wave character occur in the surface layer of a material even in its melting.

In [4], it has been shown that, in thermal destruction of the material surface, the stationary regime of removal of mass is established, once a layer whose thickness may be evaluated from the formula

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$$S(\tau_v) = \frac{d_0}{K^2_{T_\mathrm{d}}} \approx 1.83 d_0\,, \tag{1}$$

where d_0 is weakly dependant on all the properties of the material, except for its thermal conductivity [4, 5], has been removed from the material surface. In turn the stationary regime of heating is reached at the instant when the thicknesses of the heated and removed material layers are equal, i.e., when $\delta_T \approx S(\tau_\delta)$, virtually for any isotherm of the temperature field. If the isotherms with a dimensionless temperature $\theta^* < 0.1$ are taken as the heated-layer depth, this regularity, as has been established in [6], is also confirmed by calculations from the equation

$$\frac{\partial T}{\partial \tau} = a \frac{\partial^2 T}{\partial y'^2} + V_\infty \frac{\partial T}{\partial y'}\,. \tag{2}$$

At the same time, it follows from (2) that the velocities of removal of mass and the thickness of the heated layer approach their stationary values asymptotically and may attain them only at $\tau \to \infty$. In this connection, we have introduced the notions of quasistationary regimes of heating and removal of mass which are determined by the degree of approximation to the asymptote $\Delta\varepsilon = 0.1$ or 0.05 [6]. Such an approach rules out the appearance of an S-shaped temperature profile whose existence in the surface layer of the material destroyed has been proved in [7]. Such a temperature profile results from the subdivision of the temperature field into the stationary and nonstationary regions which are well described by superposition of the solutions of the heat-conduction equations with the removal of mass (2) and without the removal of mass

$$\frac{\partial T}{\partial \tau} = a \frac{\partial^2 T}{\partial y^2}\,. \tag{3}$$

For $S(\tau) < y < y_\mathrm{s}$, we obtain

$$T(y, \tau) = T_0 + (\overline{T}_\mathrm{w} - T_0) \exp\left[-\frac{\overline{V}_\infty}{a}(y - S(\tau))\right], \tag{4}$$

and for $y \geq y_\mathrm{s}$, we have

$$T(y, \tau) = T_0 + (\overline{T}_\mathrm{w} - T_0) \exp\left[-\frac{\overline{V}_\infty}{a}(y - S(\tau))\right] \mathrm{erfc}\left(\frac{y - y_\mathrm{s}}{2\sqrt{a\tau}}\right), \tag{5}$$

where $y_\mathrm{s} \approx S(\tau) + \delta_T \approx 2S(\tau)$.

Based on the results obtained in [7] we may infer that a fairly rapid attainment of the stationary velocity of movement of surface isotherms, i.e., reaching the stationary regime of heating, is responsible for the establishment of the S-shaped temperature profile in the material.

However, experimental data which make it possible to assume the presence of the S-shaped temperature profile in samples of doped quartz glass ceramics for the case where the removal of the melt film from the material surface may be disregarded, too, have been given in [8]. The conclusion drawn in [7] is, probably, not entirely responsible for such a temperature profile.

Let us assume that the attainment of such a profile in this variant of heating is a consequence of a certain wave disturbance. It is possible that this assumption will enable us to additionally explain the results given in [8]. They suggest that the initial heat flux (or its mean-integral value over the period of establishment of T_w = const) for a dimensionless removal rate of $\overline{G}_\Sigma < 0.5$ prescribes the rate of destruction of the surface irrespective of the thermal efficiency (fraction of evaporation) of the material [6]. Under different heating conditions, the rates of removal of mass are nearly equal for similar values of the mean-integral heat-flux ([8], Table 2). In the same work, it has been shown, using asbestos textolite (Fig. 10) as an example that when the calorimetric (initial) heat fluxes are equal in the stationary regime of heating, the velocities of movement of the lower boundary of the coked layer are equal, too, and do not

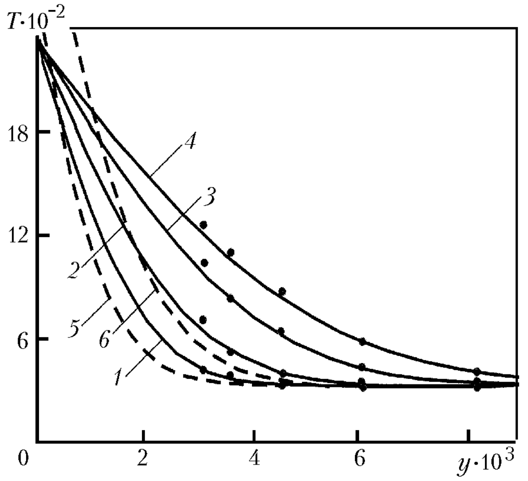


Fig. 1. Comparison of the calculated and experimental temperature profiles in a doped quartz glass ceramics at heating times $\tau \le \tau_T$: 1–4) calculation from (7) at the 2nd, 4th, 8th, and 13th second of heating at T_w = const; 5 and 6) calculation from (6) at the 2nd and 4th second of heating for q = const; points, experiment ($q_c = 2700$ kW/m^2, $T_w = 2350$ K, $a = 0.6{\cdot}10^{-6}$ m^2/sec, and $\tau_T = 13$ sec). T, K; y, m.

depend on the velocity of movement of the surface under destruction. The tests were carried out under the conditions of air and nitrogen plasmas and in radiative heating, and the rate of destruction of the surface, e.g., in the air plasma and in radiative heating, differed by three times.

In this respect, it is also of interest to consider temperature profiles obtained in samples of doped quartz glass ceramics in melting of the surface without the removal of the melt film (Fig. 1). The procedure of installation of thermocouples and the experimental procedure have been described in [8]. Figure 1 shows the calculated and experimental temperature profiles in doped quartz glass ceramics, constructed for heating times $\tau \le \tau_T$, when the surface temperature has yet to attain its constant value.

Since the calorimetric heat flux supplied to the material surface remains constant, first we use, for calculation, a formula yielded by (3) with boundary conditions of the second kind (q = const):

$$T(y, \tau) = T_0 + \frac{2q}{\lambda}\sqrt{\frac{a\tau}{\pi}}\,\exp\left(-\frac{y^2}{4a\tau}\right) - q\,\frac{y}{\lambda}\,\mathrm{erfc}\left(\frac{y}{2\sqrt{a\tau}}\right). \tag{6}$$

The heat flux entering the material considerably decreases with growth in the temperature, and the employment of its calorimetric value from (6) must ensure the higher velocity of propagation of heat compared to experimental data. The calorimetric heat-flux value used in calculations from (6) was 2700 kW/m^2.

Despite the substitution of this value of the heat flux into (6), the calculation yields much lower temperatures inside the material than those obtained by experiment (Fig. 1, curves 5 and 6). At the same time, the calculation from the formula

$$\theta^* = \frac{T^* - T_0}{T_w - T_0} = \mathrm{erfc}\left(\frac{y}{2\sqrt{a\tau}}\right) = \mathrm{erfc}\left(\frac{K}{2}\right), \tag{7}$$

which is the solution of (3) at T_w = const [9] and must hold true where the surface temperature instantaneously takes on the maximum possible value for specified heating conditions, quite satisfactorily agrees with experimental data (Fig. 1, curves 1–4).

From Eq. (7) it follows that the heated-layer thickness bounded by the isotherm with a dimensionless temperature θ^* obeys the relation

$$y \approx K \sqrt{a\tau}\,, \tag{8}$$

where the coefficient K characterizes the velocity of movement of different isotherms and is dependent just on θ^* and the law of change in $T_{\rm w}$ [10].

Since the coefficient K at $T_{\rm w}$ = const ensures the highest rate of movement of the isotherms compared to other boundary conditions, this brings up the question: in what manner is the temperature field which is immediately determined, irrespective of the prehistory of heating, by the steady-state (maximum) value of the surface temperature even at those heating times, when this temperature is not attained at all, formed? It is possible that the heat flux whose value exceeds that permissible for heating of this material without destruction prescribes the heat-penetration depth corresponding to the boundary condition $T_{\rm w}$ = const from the instant of the beginning of heating, i.e., immediately determines the maximum velocity of movement of the isotherms.

The constant calorimetric heat flux supplied to the surface decreases from its calorimetric value of $q_{\rm c}$ down to q_0' ($q_0' = q_0 - \varepsilon\sigma T_{\rm w}^4$) over the period of growth in the temperature and subsequently remains constant. The basic change occurs before the beginning of the destruction of the surface, when its temperature increases from T_0 to $T_{\rm d}$. Thereafter the surface temperature increases from $T_{\rm d}$ to $\bar{T}_{\rm w}$ over to period $\tau_T - \tau_{\rm d}$. Although this period of time may be an order of magnitude longer than $\tau_{\rm d}$ upon the attainment of which the rate of heating quite sharply changes, the heat flux entering the material decreases by no more than 10% during the period $\tau_T - \tau_{\rm d}$. We may determine the value of $\tau_{\rm d}$ in the case of a constant thermal action from the formula [6]

$$\tau_{\rm d} = \frac{\pi}{4} \lambda\rho c \frac{(T_{\rm d} - T_0)^2}{q^2}\,, \tag{9}$$

using the value of the mean-integral heat flux; it has been proposed in [8] that this heat flux be found from the equation

$$q_{\rm m}' \approx \frac{q_{\rm c}}{2}\left(1 + \frac{I_{\rm e} - I_{\rm w}}{I_{\rm e} - I_0}\right) - \frac{1}{4}\varepsilon\sigma T_{\rm w}^4\,. \tag{10}$$

It has been shown in [11] that the regularity of change in the surface temperature over the period of time from $\tau_{\rm d}$ to τ_T is determined by the thermal-destruction constant $K_{T_{\rm d}}$, and the relation

$$\tau_T \approx \left[\frac{560}{K_{T_{\rm d}}(\bar{T}_{\rm w} - T_{\rm d})} + \sqrt{\tau_{\rm d}}\right]^2 \tag{11}$$

has been proposed for evaluation of the time τ_T.

In [12], the solution of linear heat-conduction equation (3) for the initial temperature distribution $T(y, 0) = T_0 + (T_{\rm w} - T_0)\sin(\alpha y)$ in a bar of length l and the boundary conditions $T(0, \tau) = T(l, \tau) = T_0$ in the form

$$T(y, \tau) = T_0 + (T_{\rm w} - T_0)\exp(-a\alpha^2\tau)\sin(\alpha y)\,, \tag{12}$$

where $\alpha = \pi/l$, has been found.

We consider the possibility of applying (12) to explanation of the experimental data given in Fig. 2 where it is seen that the experimental values of the temperatures for y lower than a certain value of y_k are noticeably higher than those calculated from (7).

An energy diagram of thermal destruction of a material has been proposed in [3]. As a result of the accumulation of thermal energy in the surface material layer over the period from $\tau_{\rm d}$ to τ_T (area bounded by points BCD in Fig. 3), the value of K for $\theta^* = 1$ attains the value of $K_{T_{\rm d}}$ (Fig. 3, straight line 2). The temperature-field region influ-

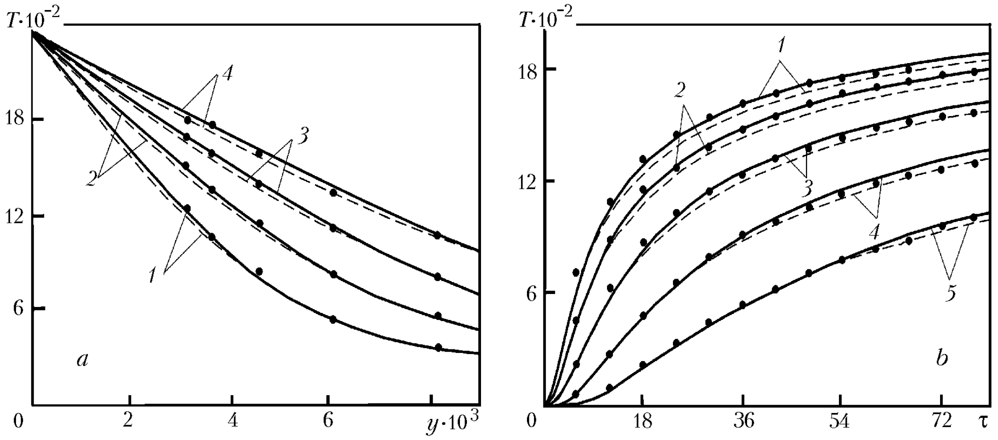


Fig. 2. Comparison of the calculated (curves) and experimental (points) values of the temperature in a doped quartz glass ceramics with allowance for the wave disturbance of the temperature field as a function of the coordinate y (a) and the heating time (b): 1–4) temperature profile at heating times of 3, 24, 42, and 70 sec; b) 1–5) at a depth of $(3.1, 3.6, 4.55, 6.05, \text{and } 8.15)\cdot 10^{-3}$ m (solid curves, calculation for $y \geq y_k$ from (14) and for $y < y_k$ from (15); dashed curves, calculation from (14), $T_w = 2350$ K, $a = 0.6\cdot 10^{-6}$ m^2/sec, and $\tau_T = 13$ sec). T, K; τ, sec.

enced by the processes in the surface layer of the material destroyed is determined by the point of intersection of dependences 1 and 2 ($\theta^* \approx 0.2$ and $K \approx 1.83$). At the steady-state surface temperature, curve 1 is calculated from (7). In turn straight line 2 for $\theta^* > 0.2$ and rates of removal of mass of $V_\infty < 0.05\cdot 10^{-3}$ m/sec in homogeneous materials is prescribed by the equation [13]

$$K = -\frac{1}{K_{T_d}}\theta^* + \frac{K_{T_d}^2}{1 - K_{T_d}} . \tag{13}$$

As has been shown in [3], if K is determined from (13) for $\theta^* > 0.2$, the quantity of thermal energy accumulated in the material and supplied to the surface is sufficient for reaching the stationary regime of removal of mass. On the other hand, if the steady-state temperature on the surface is higher than the melting temperature, and the melt film is, nonetheless, not removed from the sample's surface, the dependence $K = f(\theta^*)$ must not satisfy (13). Let us assume that in this case, too, we may represent $K = f(\theta^*)$ as a straight line for which $K = K_{T_d}$ for $\theta^* = 1$, but the value of K at the point of intersection of the straight line and dependence (7) will be less than 1.83. Then the temperature-field region y_k in which the heat accumulated in the surface material layer with increase in the temperature from T_d to T_w will be disturbed may also be evaluated from (8), where $K < 1.83$.

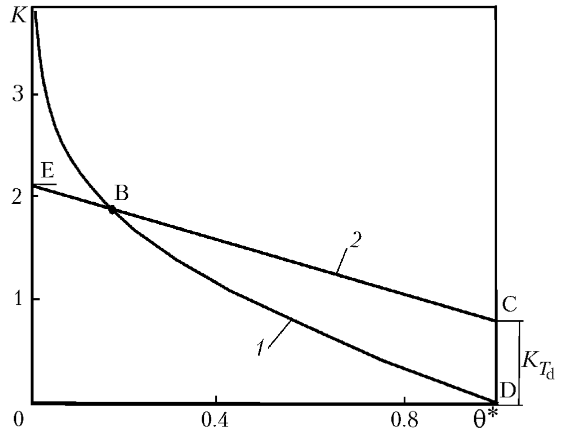
We assume that the solution (12) is applicable to approximation of the experimental data of Fig. 2 in the interval $0 < y < y_k$ for the following initial and boundary conditions: $y_k = K\sqrt{a\tau}$ at $\tau > \tau_T$, $T(0, \tau) = T_w$ at $y = 0$ and $\tau \geq \tau_T$, and $T(y_k, \tau) = T_s = \text{const}$ at $y = y_k$. Then the total approximation with account for (12) may be represented in the form of a linear superposition of two expressions:

$$T(y, \tau) = T_0 + (\bar{T}_w - T_0)\,\mathrm{erfc}\left(\frac{y}{2\sqrt{a\tau}}\right) \tag{14}$$

at $y > y_k$ and

$$T(y, \tau) = T_0 + (\bar{T}_w - T_0)\exp(-a\alpha^2\tau)\sin(\alpha y) + (\bar{T}_w - T_0)\,\mathrm{erfc}\left(\frac{y}{2\sqrt{a\tau}}\right), \quad \alpha = \pi/y_k \tag{15}$$

at $y < y_k$.



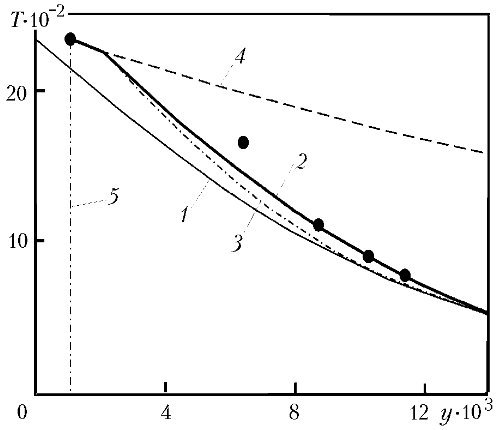


Fig. 3. Dependence of K on θ^*: 1) at $T_w < T_d$, calculation from (7); 2) at $T_w > T_d$ and in mass removal, calculation from (13); the values of K at points C, B, and E are 0.74, 1.83, and 2.09 respectively.

Fig. 4. Comparison of the calculated and experimental temperature profiles in a doped quartz glass ceramics at the 60th second of heating for $V_\infty = 0.02 \cdot 10^{-3}$ m/sec, $T_w = 2350$ K, $a = 0.65 \cdot 10^{-6}$ m^2/sec, and $\tau_v = 56$ sec: 1) calculation from (7), 2) from (17), 3) from (5) for $y \geq y_s$; 4) from (4) for $S(\tau) < y < y_s$; and 5) position of the surface under heating; points, experiment. T, K; y, m.

A comparison of the experimental and calculated values of the temperatures from (14) and (15) shows that the coefficient K in (7) must be equal to 1.6. The calculated data satisfactorily agree with experiment for this value of K. From formulas (9), (10), and (11), we find values of $\tau_d \approx 2$ sec and $\tau_T \approx 13$ sec. In calculating, we used the following values of the parameters: $q_c = 2700$ kW/m^2, $T_w = 2350$ K, $T_d = 2000$ K, $a = 0.6 \cdot 10^{-6}$ m^2/sec, and $\rho c = 2530$ kJ/(m^3·K).

Since (14) and (15) have been obtained at T_w = const, we should compare the calculated and experimental data for $\tau > \tau_T$. From Fig. 2 it is clear that the dependences (calculated from (14) and (15)) of the temperature on the heating time and the coordinate y satisfactorily agree with experimental results. The half-wave amplitude established in the region of the temperature field $y < y_k$ has its maximum value for $y_k/2$ and gradually decreases with increase in the heating time, unless the removal of mass from the material surface begins. It is noteworthy that the experimental temperature field in the sample of quartz glass ceramics, probably, more rapidly turns back to the self-similar regime of heating (14) than follows from the calculation from (15) (Fig. 2, $\tau > 70$ sec).

As the density of the heat flux supplied increases further, the removal of the melt film from the sample's surface begins. Its influence on the temperature field for $V_\infty < 0.05 \cdot 10^{-3}$ m^2/sec propagates to the isotherm with a dimensionless temperature $\theta^* \approx 0.2$ [8]. The dependence $K = f(\theta^*)$ at $\theta^* > 0.2$ is described according to (13), and K attains a value of 1.83 (Fig. 3), since in this case the stationary regime of removal of mass from the material surface must earlier or later be established (when $K < 1.83$ it is virtually absent and the wave disturbance is attenuated). In mass removal, a half-wave which, probably, results from the action of the heat flux exceeding a level permissible for heating a material without destruction, is one reason for the reduction in the time of reaching the stationary regime of removal of mass; according to [4], it is determined from the formula

$$\tau_v = \frac{K_{T_d}^2 + 1}{K_{T_d}^2} \frac{d_0}{\overline{V}_\infty}. \tag{16}$$

To calculate the temperature field for $S(\tau) < y < y_s$ we may use relation (4). However, for $y \geq y_s$, we must allow for the wave term in (5) by analogy with (15):

$$T(y, \tau) = T_0 + (\overline{T}_w - T_0) \exp(-a\alpha^2\tau) \sin[\alpha(y - y_s)] +$$

$$+ (\bar{T}_{\mathrm{w}} - T_0) \exp\left[-\frac{\bar{V}_\infty}{a}(y - S(\tau))\right] \operatorname{erfc}\left(\frac{y - y_{\mathrm{s}}}{2\sqrt{a\tau}}\right), \tag{17}$$

where $y_k = 1.83\sqrt{\alpha\tau}$ and $S(\tau) = \bar{V}_\infty\tau - d_0$.

Figure 4 compares the calculated and experimental temperature profiles in the sample of quartz glass ceramics ($a = 0.65 \cdot 10^{-6}$ m^2/sec) for the heating time $\tau > \tau_v$. It is clear form the figure that the calculation from (17) with allowance for the wave disturbance is in better agreement with experimental data than the calculation from (5).

CONCLUSIONS

1. Under conditions where the heat flux exceeds a value permissible for heating a material without destruction, the velocity of propagation of heat is independent of the prehistory of heating and, from the beginning of thermal action, is determined by the steady-state value of the temperature of the surface being destroyed.

2. It has been shown that a wave disturbance of the temperature field is possible under such heating conditions. The maximum amplitude of the half-wave does not exceed 70 K in the case considered and is attenuated with increase in the heating time. At the same time, the wave disturbance is, probably, one reason for the acceleration of reaching the stationary regime of destruction of the surface in mass removal.

3. The results obtained confirm that any dissipative structure, including the nonstationary heated layer of material, when the heat flux supplied exceeds the critical value, transforms into a new, dynamically stable stationary state as a result of fluctuation.

NOTATION

a, thermal diffusivity, m^2/sec; c, heat capacity, kJ/(kg·K); d_0, parameter of nonstationary removal of mass, m; $\bar{G}_\Sigma$, dimensionless rate of removal of mass; I_{e}, stagnation enthalpy, kJ/kg; I_{w} and I_0, enthalpy of the gas at the hot- and cold-surface temperatures, kJ/kg; K, coefficient characterizing the velocity of movement of the isotherm; $K_{T_{\mathrm{d}}}$, thermal-destruction constant; l, bar length, m; q, heat flux, kW/m^2; q_0, convective heat flux to the hot impermeable surface, kW/m^2; q_{c}, heat flux to the cold calorimetric surface, kW/m^2; $S(\tau)$, $S(\tau_v)$, and $S(\tau_\delta)$, linear removal and its values at the instant of attainment of the stationary values of the removal rate and the heated-layer thickness, m; T, temperature, K; T_0, temperature of the unheated layer, K; T_{s}, temperature of the lower boundary of the zone of wave disturbance of the temperature field (y_k), K; T_{w} and T_{w}, temperature of the surface under heating and its steady-state value, K; T^*, temperature of the isotherm, K; $T(y)$, running value of temperature, K; T_{d}, temperature of destruction (melting) of the material surface, K; V_∞ and V_∞, rate of linear removal of mass and its stationary value, m/sec; y and y', coordinates counted off from the initial and moving surfaces, m; y_{s}, coordinate of the lower boundary of the stationary heated layer; it is counted off from the initial surface, m; y_k, coordinate of the lower boundary of the wave disturbance of the temperature field in melting without the removal of mass, m; δ_T, stationary value of the heated-layer depth, m; $\Delta\varepsilon$, degree of approximation to the asymptote; θ^*, dimensionless temperature of the isotherm; λ, thermal conductivity, W/(m·K); ρ, density, kg/m^3; ε, emissivity factor of the surface; σ, Stefan–Boltzmann constant, kW/(m^2·K^4); τ, heating time, sec; τ_T, τ_v, and τ_δ, times of establishment of the stationary values of the surface temperature, the mass-removal rate, and the heated-layer thickness, sec; τ_{d}, time of the beginning of the destruction (melting) of the surface, sec. Subscripts: 0, unheated material, impermeable surface; e, external boundary of the boundary layer; k, zone of wave disturbance of the temperature field; v, velocity; w, wall conditions; Σ, total; δ, heated layer; c, calorimetric; d, destruction; m, mean-integral; s, stationary.

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